A data-driven industrial alarm decision method via evidence reasoning rule

Xu Weng\textsuperscript{a,}Xiaobin Xu\textsuperscript{a,}\textsuperscript{*}, Yu Bai\textsuperscript{a,b}, Feng Ma\textsuperscript{c}, Guodong Wang\textsuperscript{d}, Schahram Dustdar\textsuperscript{e}

\textsuperscript{a}School of Automation, Hangzhou Dianzi University, Hangzhou 310018, China
\textsuperscript{b}Zhejiang Academy of Traditional Chinese Medicine, Hangzhou 310012, China
\textsuperscript{c}Nanjing Smart Water Transport Technology Co., Ltd., Nanjing 210000, China
\textsuperscript{d}Shanghai Institute of Computing Technology, Shanghai 200000, China
\textsuperscript{e}Distributed Systems Group, TU Wien, 1040 Vienna, Austria

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\textbf{ABSTRACT}

In order to deal with the generalized uncertainty of the process variable, under the framework of Dempster–Shafer theory of evidence (DST), a data-driven approach without any probabilistic assumption is presented via the dynamic form of the evidence reasoning (ER) rule. Firstly, the process variable is transformed into the corresponding alarm evidence according to referential evidential matrix constructed by casting historical samples. Secondly, the ER rule is proposed to recursively combine the current and historical alarm evidence to generate the global alarm evidence for alarm decision. In the process of recursive fusion, the forgetting strategy is introduced to calculate the reliability factors of the current and historical alarm evidence; the genetic algorithm is designed to optimize the importance weights of evidence. Finally, numerical experiment and industrial case are given to show that the proposed method has a better performance than the classical methods and the initial conditional evidence updating method.

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1. Introduction

In the design of industrial alarm systems, benefiting from the application of many technologies such as feature extraction, pattern classification and information fusion in fault diagnosis, the main process variables in industrial production lines or large equipment can be effectively monitored and processed \cite{1,2}. By detecting the abnormal conditions of these process variables, operators or maintenance engineers then adopt measures such as degraded operation or emergency shutdown to troubleshoot the accidents, prevent the abnormal condition from spreading widely and even eliminate failures \cite{3,4}. Hence the alarm system is an important means to ensure the safety of industrial production and maintain the safe and stable operation of equipment. In the univariate alarm design problem, one of the most common design methods is to compare the sample values of the process variable with a designed alarm threshold \cite{5,6}. If the sample values exceed alarm threshold, an alarm will be activated. However, due to the increasing complexity of industrial systems, this most common alarm design method has high false alarms, frequent alarm overload, therefore there is still a huge possibility for improvement \cite{7}.

In recent years, the scholars have proposed some alarm optimization design methods based on dead band, time delay and filtering to improve the performance of univariate alarm \cite{8–10}. A general idea of these methods is to do data preprocessing before comparing the sample values of the process variable with alarm threshold. At the same time, with the continuous development of research in the field of alarm management, some authoritative industry standards have proposed three main performance evaluation indices of alarms: false alarm rate (FAR), missed alarm rate (MAR), and average alarm delay (AAD) \cite{11,12}. Here, the false alarm rate and the missed alarm rate measure the accuracy of alarm, and the average alarm delay measures the sensitivity of alarm. Accuracy and sensitivity together constitute the core of the alarm performance evaluation, becoming an important factor that should be considered at the beginning of alarm system design. By using FAR/MAR/AAD and the receiver operating characteristic curve (ROC), the relevant parameters (alarm threshold, filtering order, delay steps, etc.) of the above alarm design methods are optimized so as to further improve the performance of univariate alarm \cite{13}.

Essentially, these univariate alarm design methods provide different types of classifiers to distinguish the normal state and abnormal state of the process variable with generalized uncertainty. The generalized uncertainty is a unified description of the aleatory uncertainty \cite{14}, the epistemic uncertainty \cite{15} and the
mixtures of these two uncertainties. In the process of classifier design, the probability distributions of the process variables need to be first obtained through statistical analysis of historical data, and then the parameters of the classifiers are optimized by using probabilistic methods. However, in real industrial production, facing various uncertain factors such as power frequency interference, electromagnetic interference and irregular mechanical vibration, and the limited understanding for the complex operating processes of industrial systems, it becomes more and more difficult to obtain precise probability distributions of the main process variables. Hence, it seems no longer appropriate to design alarm system only depending on the probability theory. At the same time, other different theories are used to accurately describe the generalized uncertainty and comprehensively improve the performance of the industrial alarm. Our earliest research work studied linear recursive updating rules of the conditional evidence via Dempster–Shafer theory of evidence (DST) and explored the effectiveness of evidence updating tactics on improving the accuracy of industrial alarm [16].

Moreover, in order to explore alternatives to handle the generalized uncertainty of the process variables, based on the dynamic form of evidence reasoning (ER) rule in DST, this paper proposes a pure data-driven alarm design method regardless of any probability hypothesis. To be more specific, the monitoring value of the process variable is transformed into the corresponding probability hypothesis. To be more specific, the monitoring value of the process variable is transformed into the corresponding probability hypothesis. Thus, the alarm evidence is a belief distribution about propositions or hypotheses “Alarm (A)” and “Non-Alarm (NA)”. The dynamic form of ER rule is presented to recursively combine the current and historical alarm evidence to generate the global alarm evidence. The recursive fusion process can integrate more alarm information to effectively reduce generalized uncertainty in the original process variable. Hence, the alarm decision based on the global alarm evidence will be more objective and reliable than those based on the original variable or the single current/historical evidence. In addition, the forgetting strategy in CBR methodology is introduced to calculate the reliability factors of the current and historical alarm evidence according to the similarity measure between two pieces of evidence. And the genetic algorithm is designed to optimize the importance weights of corresponding alarm evidence.

This paper is organized as follows: Section 2 introduces the main performance indicators of industrial alarm and alarm decision; Section 3 gives the theoretical basis of DST and evidence reasoning rule; Section 4 proposes the presented alarm design process based on the dynamic ER rule; in Section 5, a numerical example and a real industrial case are given to illustrate performance of the proposed ER-based data-driven method and its advantages over the traditional alarm design method based on probability theory and the initial conditional evidence updating method. And conclusion is made in Section 6.

2. Performance indices and alarm decision

Let \(x(t)\) be a discrete sample of the process variable \(x\) with a sampling period \(h\). \(x(t)\) has two states: “normal state” and “abnormal state”. By comparing with the alarm threshold (trip point) \(x_{tp}\), if \(x(t) \geq x_{tp}\), an alarm is generated, if \(x(t) < x_{tp}\), there is no alarm. In this process, there are two undesired situations, namely, a false alarm in the normal state and a missed alarm in the abnormal state. As shown in Table 1, the two main performance indices of the alarm system: the false alarm rate (FAR) and the missed alarm rate (MAR), are derived from the corresponding relations between the false (missed) alarms of the alarm system and the normal (abnormal) states of the process variables [17].

<table>
<thead>
<tr>
<th>Hypothesized classes</th>
<th>True classes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>Non-alarm</td>
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</tbody>
</table>

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Alarm</td>
<td>TN</td>
<td>FA</td>
<td>MA</td>
<td>TA</td>
</tr>
</tbody>
</table>

Accordingly, as \(x_{tp}\) changes, the entries (TN, MA, FA and TA) all change, and their sum is equal to the length of the sample sequence \((x(1h), x(2h), \ldots)\). FAR and MAR are defined as follows [18]

\[
\text{FAR} = \left(\frac{FA}{FA + TN}\right) \times 100\% \tag{1}
\]

\[
\text{MAR} = \left(\frac{MA}{TA + MA}\right) \times 100\% \tag{2}
\]

By plotting receiver operating characteristic curve (ROC), FAR and MAR will change with the values of \(x_{tp}\). The optimal alarm threshold \(x_{tp}\) usually refers to that corresponding to the point closest to the origin (FAR = 0%, MAR = 0%) on the ROC curve. If the time when the abnormality occurs is \(t_0\) and the time when the corresponding alarm occurs is \(t_a\), the alarm delay time \(T_a\) can be calculated as

\[
T_a = t_a - t_0 \tag{3}
\]

If there are \(N\) sequences of samples, then \(N\) delay times \(T_{d1}, T_{d2}, \ldots, T_{dN}\) can be obtained. Then the average alarm delay (AAD) can be defined as [19]

\[
\text{AAD} = \left(\frac{T_{d1} + T_{d2} + \cdots + T_{dN}}{N}\right) \tag{4}
\]

3. Theoretical basis

3.1. The basic concepts in DST

A frame of discernment \(\Theta = \{H_1, H_2, \ldots, H_n\}\) is a collection of mutually exclusive hypotheses, \(\Theta\) and all of its subsets are together called a power set denoted as \(P(\Theta)\) or \(2^\Theta\).

**Definition 3.1**[20]. A mapping operation \(m: 2^\Theta \rightarrow [0, 1]\) is a basic belief assignment function (BBA) on \(\Theta\) and satisfies \(m(\emptyset) = 0\) and \(\sum_{Y \subseteq \Theta} m(Y) = 1\).

\(m(\emptyset)\) represents the belief assignment of the proposition \(\emptyset\), and a basic belief assignment function obtained from certain information source is called a body of evidence or a piece of evidence abbreviated to evidence. For the design of the alarm system, the proposition \(\theta = \{A, NA, \Theta\}\), here \(\Theta = \{A\) (Alarm), NA (Non-Alarm)\).

**Definition 3.2**[21]. Evidence distance between the two pieces of evidence \(m_1\) and \(m_2\) is defined by Jousselme as follows

\[
d_J(m_1, m_2) = \sqrt{\frac{1}{2} \left( \sum_{D \in 2^\Theta} D(m_1 - m_2)^2 \right)} \tag{5}
\]

where \(D\) is a \(n \times n\) matrix, \(n\) is the number of non-empty sets elements in the power set \(P(\Theta)\), its elements \(D(A, B) = |A \cap B|/|A \cup B|, A, B \in 2^\Theta\), called the Jaccard coefficient. In the context of alarm system design, a piece of alarm evidence (BBA) can be modeled by \(m = (m(A), m(NA), m(\emptyset))\), where the order of \(D\) is \(n = 3\), that is

\[
D = \begin{bmatrix}
1 & 0 & 1/2 \\
0 & 1 & 1/2 \\
1/2 & 1/2 & 1
\end{bmatrix} \tag{6}
\]
When $d_1(m_1, m_2) = 0$, it means that $m_1$ and $m_2$ are exactly same, and $d_1(m_1, m_2) = 1$ means that both of them are completely opposite. The similarity between $m_1$ and $m_2$ can be defined as:

$$\text{Sim}(m_1, m_2) = 1 - d_1(m_1, m_2)$$  \hfill (7)

Obviously, the larger $d_1(m_1, m_2)$, the smaller $\text{Sim}(m_1, m_2)$, and the value range of the similarity is $[0, 1]$.

### 3.2. Evidence reasoning (ER) rule

The newly proposed evidence reasoning (ER) rule based on the orthogonal sum theorem follows a strict probabilistic reasoning process and clearly distinguishes the conceptions of evidence reliability and importance [22]. A piece of evidence in the ER rule can be expressed as follows

$$e = \{ (\theta, m(\theta)) | \forall \theta \subseteq \Theta, \sum_{\theta \subseteq \Theta} m(\theta) = 1 \}$$  \hfill (8)

Wherein the element $(\theta, m(\theta))$ of evidence $e$ indicates that the degree of support for the proposition $\theta$ is $m(\theta)$, the proposition $\theta$ can take any element other than the empty set in $2^\Theta$, here $m(\theta)$ is representation of the BBA function in Definition 3.1. If $m(\theta) > 0$, then $\theta$ is called the focal element of $e$. Obviously, here the BBA function $m$ reflecting the relationship between the proposition and its belief is expressed as evidence $e$ with the form of number pairs.

In ER rule, $e$ is associated with a reliability factor $r$ and an importance weight $w$. The $r$ objectively characterizes the ability of evidence $e$ that provide an accurate assessment or answer to a given question. The $r$ is an intrinsic property of evidence, and will not be affected by other evidence; the $w$ subjectively focuses on the corresponding importance of $e$ relative to other evidence. The belief distribution function with reliability factor and importance weight is defined as follows

$$\hat{e} = \{(\theta, \hat{m}(\theta)) | \forall \theta \subseteq \Theta; (P(\theta), \hat{m}(P(\theta)))]$$  \hfill (9)

Wherein, $\hat{m}(\theta)$ considering $r$ and $w$ represents the degree of support for $\theta$

$$\hat{m}(\theta) = \begin{cases} 1 & \theta = \emptyset \\ c m(\theta) & \emptyset \subseteq \theta, \theta \neq \emptyset \\ c (1-r) & \theta = P(\theta) \end{cases}$$  \hfill (10)

here, $m(\theta) = w m(\theta)$ is a normalization factor so that $\sum_{\theta \subseteq \Theta} \hat{m}(\theta) + \hat{m}(P(\theta)) = 1$.

For two independent evidence $e_1$ and $e_2$, the corresponding reliability factors and importance weights are $(r_1, r_2)$ and $(w_1, w_2)$ respectively, and they can be fused by ER rule, as shown below

$$m(\theta)^{e^{(2)}} = \begin{cases} 1 & \theta = \emptyset \\ \hat{m}(\theta)^{e^{(2)}} & \emptyset \subseteq \theta, \theta \neq \emptyset \\ \hat{m}(\theta)^{e^{(2)}} + (1 - r_1)m(\theta)_1 + (1 - r_2)m(\theta)_2 & \emptyset \subseteq \theta \end{cases}$$  \hfill (11)

$$\hat{m}(\theta)^{e^{(2)}} = \begin{cases} 1 & \theta = \emptyset \\ \hat{m}(\theta)^{e^{(2)}} & \emptyset \subseteq \theta, \theta \neq \emptyset \\ \hat{m}(\theta)^{e^{(2)}} + (1 - r_1)m(\theta)_1 + (1 - r_2)m(\theta)_2 & \emptyset \subseteq \theta \end{cases}$$  \hfill (12)

It can be used recursively for combining multiple pieces of evidence in any order.

### 4. Alarm design method based on ER rule and forgetting strategy

Here, Section 4.1 will detail how to transform $x(t)$ into the alarm evidence $m_1 = (m_1(A), m_1(NA), m_1(\Theta))$ by referential evidential matrix (REM) constructed by casting historical samples. In Section 4.2, the current alarm evidence $m_1$ and the historical global alarm evidence $m_{1:t-1}$ containing all previous information are recursively combined by ER rule to obtain the current global alarm evidence $m_{1:t}$ for alarm decision. In addition, considering that the relationship between the new alarm evidence and the historical alarm evidence should not be given or static, the reliability factor $r$ and the importance weight $w$ of the corresponding evidence may change with the sampling time. Therefore, when using ER rule for recursive iterative fusion, the selection of two parameters $(r, w)$ should satisfy this dynamic situation. So in Section 4.3, a method of using the forgetting strategy to dynamically calculate the reliability factors of current and historical alarm evidence is proposed. And Section 4.4 shows how to use historical samples to construct genetic algorithm to obtain the optimal importance weights of historical global alarm evidence and current alarm evidence. At the end of Section 4.4, a flowchart is exhibited to illustrate the process of parameter selection and dynamic alarm evidence updating, and clarify the assumptions of the proposed alarm design method.

### 4.1. Alarm evidence generation method based on historical samples

In the classical alarm generation mechanism, it is generally decided whether to alarm directly by judging whether process variable $x(t)$ absolutely exceeds alarm threshold $x_p$, this design idea lacks a more general expression and flexible description of generalized uncertainty of the process variable. This section proposes an alarm evidence generation method through statistical analysis of historical samples to construct a referential evidential matrix, so as to the sample values of $x(t)$ are converted into the corresponding alarm evidence.

Let the historical samples set be $Z = \{ x(t), y(t) \}$, wherein $x(t)$ and $y(t)$ represent the input and output of the designed alarm respectively. The reference values set of input $x(t)$ is $R = [R_i | i = 1, 2, \ldots, I]$, $I$ is the number of reference values, and analogously the reference values set of output $y(t)$ is $Y = \{ Y_j | j = 1, 2 \}$, where $Y_1 = NA$ and $Y_2 = A = 1$. The reference values of the input $x(t)$ can be given according to expert knowledge, and the reference values 0 and 1 of the output $y(t)$ respectively represents two states, namely, non-alarm state ‘‘NA’’ and alarm state ‘‘A’’. Thus, the relationship between the input $x(t)$ and output $y(t)$ is approximately converted into the relationship between the input reference values and the output reference values. For the input $x(t)$ of the historical samples set $Z = \{ x(t), y(t) \}$, its similarity distribution about the reference values set $R$ can be obtained by the following information conversion technique.

$$V_0(x(t)) = \langle [R_i, \alpha_i] | i = 1, 2, \ldots, I \rangle$$  \hfill (13)

among them

$$\alpha_i = (R_{i+1} - x(t))/(R_{i+1} - R_i)$$  \hfill (14)

$$\alpha_{i+1} = (x(t) - R_i)/(R_{i+1} - R_i)$$  \hfill (15)

$\alpha_i$ represents the similarity of the input $x(t)$ matching the reference value $R_i$, $\alpha_{i+1}$ represents the similarity of the input $x(t)$ matching the reference value $R_{i+1}$, obviously, $\alpha_i + \alpha_{i+1} = 1$.

The similarity distribution of the output $y(t)$ matching reference values $Y_j$ is

$$V_j(y(t)) = \langle [Y_j, \lambda_j] | j = 1, 2 \rangle$$  \hfill (16)

among them

$$\lambda_j = \begin{cases} 1 & Y_j = NA \\ 0 & Y_j = A \end{cases}$$  \hfill (17)

$$\lambda_{j+1} = 1 - \lambda_j$$  \hfill (18)
Table 2
Sample reference value cast point table.

<table>
<thead>
<tr>
<th>( y(t) )</th>
<th>( x(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{R}_1 )</td>
<td>( \delta_{1,1} ) ( \delta_{1,2} ) ( \delta_{1,3} ) ... ( \delta_{1,J} ) ( \psi_1 )</td>
</tr>
<tr>
<td>( \tilde{R}_2 )</td>
<td>( \delta_{2,1} ) ( \delta_{2,2} ) ( \delta_{2,3} ) ... ( \delta_{2,J} ) ( \psi_2 )</td>
</tr>
<tr>
<td>Total</td>
<td>( o_1 ) ( o_2 ) ( o_3 ) ... ( o_J ) ( K )</td>
</tr>
</tbody>
</table>

\( \lambda_j \) represents the similarity of the output \( x(t) \) matching the reference values \( Y_j \).

Thus, each group of samples in the historical samples set \( Z = \{ x(t), y(t) \} \) is transformed into the form of the comprehensive similarity distribution \( (\alpha_{\lambda_j}, \alpha_{\lambda_j+1}, \alpha_{\lambda_j+1}, \alpha_{\lambda_j+1}, \alpha_{\lambda_j+1}) \). Obviously, \( \alpha_{\lambda_j} + \alpha_{\lambda_j+1} + \alpha_{\lambda_j+1} + \alpha_{\lambda_j+1} = 1 \). Where \( \alpha_{\lambda_j} \) represents the comprehensive similarity of the input \( Z \) matching the reference value \( R_i \), and the output \( y(t) \) matching the reference value \( Y_j \). The cast points of all samples in the historical samples set \( Z \) are calculated, and then a sample reference value cast point table that characterizes the relationship between the input reference value \( R_i \) and the output reference value \( Y_j \) is established, as shown in Table 2. Where \( \delta_{ij} \) represents the sum of the similarity of the samples \( \{ x(t), y(t) \} \) whose input \( x(t) \) matches the reference value \( R_i \) and the output \( y(t) \) matches the reference value \( Y_j \),

\[
\psi = \sum_{i=1}^{M} \delta_{ij} \]

\( \psi \) represents the sum of the overall similarity of \( y(t) \) matching \( Y_j \) in all samples, \( 0 = \sum_{j=1}^{N} \delta_{ij} \) represents the sum of the overall similarity of \( x(t) \) matching \( R_i \) in all samples, and there is \( \sum_{j=1}^{N} \psi = \sum_{i=1}^{M} \alpha_{ij} = K \).

According to Table 2, when the input \( x(t) \) takes the reference value \( R_i \), the belief that the output \( y(t) \) takes the reference value \( Y_j \) is

\[
\tilde{e}_{ij} = \frac{\psi_{ij}}{\psi} \sum_{j=1}^{N} (\delta_{ij} \psi_{ij})
\]

and there is \( \sum_{j=1}^{N} \tilde{e}_{ij} = 1 \), then the alarm evidence corresponding to the reference value \( R_i \) can be defined as \( e_i \)

\[
e_i = (\{ NA, \tilde{e}_{ij} \}, \{ A, \tilde{e}_{ij} \})
\]

Thus, each newly measured value \( x(t) \), an alarm evidence \( m_t = (m_t(A), m_t(NA), m_t(\Theta), m_t(\emptyset)) \) can be obtained, where \( m_t(\Theta) = 0 \), \( m_t(NA) \) and \( m_t(A) \) are given by formulas (21) and (22) respectively. According to this conversion mechanism, the \( x(t) \) containing the original information are converted into the corresponding alarm evidence \( m_t = (m_t(A), m_t(NA), m_t(\Theta), m_t(\emptyset)) \). Note that the proposed conversion mechanism is a purely data-driven method that does not require assumption and modeling of the relationship between “Alarm (A)” and “Non-Alarm (NA)”, and this non-destructive conversion process from original information to alarm evidence is critical for the subsequent fusion of alarm evidence.

4.2. Recursive fusion of alarm evidence based on ER rule

According to Section 4.1, the current alarm evidence \( m_t \) about the process variable \( x(t) \) can be obtained. Then, the ER rule (formula (11)) in Section 3.2 is extended to the designed alarm to realize the iterative fusion of corresponding alarm evidence. That is, the current alarm evidence \( m_t \) is fused with the historical global alarm evidence \( m_{t-1} \), and the current global alarm evidence \( m_{t+1} \) is recursively obtained. Combining expert knowledge, the initial values of important weight \( w_{t+1} \) of \( m_{t+1} \) and \( w_{t} \) of \( m_{t} \) need to be optimized to fully characterize the dynamic relationship between historical global alarm evidence and current alarm evidence. The next two sections are presented to achieve these dynamic change of \( r \) and \( w \) to improve the accuracy of the alarm.

4.3. The calculation of the reliability factors of alarm evidence based on forgetting strategy

In the process of dynamic fusion of corresponding evidence in Section 4.2, the fusion parameters \( r, w \) are determined according to expert knowledge. In addition, since the alarm design method based on ER rule adopts the incremental learning
method, as the time $t$ increases, the global alarm evidence $m_{1,t}$ obtained after the process of recursive fusion will tend to a certain result without restriction, which has been make the “Alarm (A)” or “Non-Alarm (NA)” decision falsely. The excessive iterative fusion of the historical alarm evidence $m_{1,t-1}$ at the redundant time and the current alarm evidence $m_t$ will also cause the overall sensitivity of the alarm to decrease. Therefore, it is also necessary to realize the rational selection of the fusion parameters $(r, w)$, when subjectively given fusion parameters result in poor performance of the designed alarm, more options are provided for the operator so as to obtain better alarm performance.

This section first introduces a calculation method for the reliability factor of the new and historical alarm evidence. The reliability factor $r$ is the inherent nature of the alarm evidence itself and will not be affected by other alarm evidence. Hence, based on the forgetting strategy (FS) in case-based reasoning (CBR) [23,24], the idea of intentional forgetting is introduced into the iterative process of alarm evidence fusion. That is, the reliability factor of current and historical alarm evidence is calculated according to the similarity measure between the two pieces of evidence. This adjustment of reliability factor achieves dynamic real-time changes in the relationship between the new and historical alarm evidence.

Therefore, the reliability factor $r_t$ of the current alarm evidence $m_t$ is calculated by the following formula:

$$r_t = \tilde{r} + \tau \cdot \phi \cdot r_0 \quad r_t \in [0, 1]$$  \hspace{1cm} (27)

Among them, $\tilde{r}$ is the average value of the reliability factor of the alarm evidence at the fast $t$ time, the value of $l$ can be determined according to the sample size. And the reliability factor of the historical global alarm evidence $m_{1,t-1}$ is also given by $\tilde{r}$.

$$r_{1:t-1} = \tilde{r} = \frac{1}{l} \sum_{t=1}^{l} r_{t-1} \quad r_{t-1} \in [0, 1]$$  \hspace{1cm} (28)

$r_0$ is the initial value of reliability factor with adjustable parameters, generally $r_0$ is set to 0.5;

$\tau$ is the reward and punishment factor, by comparing the current alarm evidence $m_t$ with the historical global alarm evidence $m_{1,t-1}$, if the two alarm evidence points to the same proposition, the value is set to 1, otherwise, the value is set to $-1$, which can be calculated by

$$\tau = \begin{cases} 
1, & (m_t(NA) > m_t(A)) \land (m_{1,t-1}(NA) > m_{1,t-1}(A)) \\
& \land (m_t(NA) < m_t(A)) \land (m_{1,t-1}(NA) < m_{1,t-1}(A)) \\
-1, & (m_t(NA) > m_t(A)) \land (m_{1,t-1}(NA) < m_{1,t-1}(A)) \\
& \land (m_t(NA) < m_t(A)) \land (m_{1,t-1}(NA) > m_{1,t-1}(A)) 
\end{cases}$$  \hspace{1cm} (29)

$\phi$ is a reliability enhancement factor based on the similarity measure between the two pieces of evidence, calculated by

$$\phi = \frac{1 - d_2}{1 - d_1 + (1 - d_2)}$$  \hspace{1cm} (30)

Among them, $d_1$ and $d_2$ are the historical global alarm evidence $m_{1,t-1}$ and the current alarm evidence $m_{1,t}$, which are respectively combined with the standardized evidence $(e_{NA} = (1, 0, 0), e_t = (0, 1, 0))$ to calculate the evidence distance by using Definition 3.2, given by the following formula:

$$d_1 = \begin{cases} 
\sqrt{\frac{(m_{1,t-1} - e_{NA}) \cdot D \cdot (m_{1,t-1} - e_{NA})^T}{2}}, & m_{1,t-1}(NA) > m_{1,t-1}(A) \\
\sqrt{\frac{(m_{1,t-1} - e_t) \cdot D \cdot (m_{1,t-1} - e_t)^T}{2}}, & m_{1,t-1}(NA) \leq m_{1,t-1}(A) 
\end{cases}$$  \hspace{1cm} (31)

$$d_2 = \begin{cases} 
\sqrt{\frac{(m_t - e_{NA}) \cdot D \cdot (m_t - e_{NA})^T}{2}}, & m_t(NA) > m_t(A) \\
\sqrt{\frac{(m_t - e_t) \cdot D \cdot (m_t - e_t)^T}{2}}, & m_t(NA) \leq m_t(A) 
\end{cases}$$  \hspace{1cm} (32)

Through the above dynamic adjustment of the reliability factor of current and historical alarm evidence, the relationship of corresponding evidence during the recursive iterative fusion process is guaranteed to be updated in real time, and the efficiency of dynamic fusion of evidence is also improved. Compared to fixed reliability factors that is subjectively given by expert knowledge, this calculation of the reliability factors can play a positive role in reducing the FAR and MAR of the designed alarm obviously.

4.4. The optimization of the importance weight of alarm evidence

In the previous section, the dynamic adjustment of the reliability factor improves the performance of the alarm. In this section, the importance weight $w$ is adjusted. Compared with the reliability factor $r$, which needs to be updated in real time during each fusion process, the importance weights $w_1$ and $w_2$ of the historical global alarm evidence $m_{1,t-1}$ and the current alarm evidence $m_t$ subjectively define the relative importance of the alarm evidence compared to other alarm evidence. It is determined by what kind of evidence participates in the dynamic fusion, the users of the evidence, and the specific use cases of the evidence. Therefore, based on the calculation of the reliability factor $r$, the sample set $W$ is used to train the importance weight $w$ in order to improve the performance of the model. Under the condition that the model is reasonable, the evidence distance between the dynamic fusion result $m_{1,t}$ and the standardized evidence $(e_{NA} = (1, 0, 0), e_t = (0, 1, 0))$ is used as the objective function, and the optimization model based on genetic algorithm is constructed as follows

$$\min_{\xi} \xi(G) = \sum_{t=1}^{l} d_t(m_{1,t}, e^t)$$  \hspace{1cm} (33)

s.t. \hspace{0.5cm} 0 \leq w_i \leq 1, \hspace{0.5cm} i = 1, 2$$

where $\xi(G)$ represents the optimization objective function, $G = \{w_i | i = 1, 2\}$ represents the parameter set to be optimized, and the value range of $w_i$ is $[0,1]$. The $d_t$ indicates the fusion result, that is, the evidence distance between the global alarm evidence $m_{1,t}$ and the standardized evidence $(e_{NA} = (1,0,0), e_t = (0,1,0))$. This optimization process can be done through the global optimization toolbox in Matlab. Finally, the performance of the model is gradually optimized due to the change of the optimized parameter set $G$.

To summarize, Fig. 1 shows the process of parameter selection and dynamic alarm evidence updating. Note that the optimal importance weights $w_1$ and $w_2$ of historical global alarm evidence $m_{1,t-1}$ and current alarm evidence $m_t$ need to be obtained based on historical data, which is the training process of the ER-based industrial alarm system. The reliability factors of current and historical alarm evidence are obtained by dynamic calculation. All of the selection of these two parameters are ultimately used to obtain the optimal ER-based industrial alarm system and make appropriate alarm decision. In addition, it is necessary to summarize the assumptions of the proposed alarm design method: (1) design of univariate alarm systems; (2) the types of uncertainty in process variable $x(t)$ include the random uncertainty, the epistemic uncertainty and the mixtures of these two uncertainties; (3) the $x(t)$ obeys the independent and identically distributed (IID) or non-IID; (4) focus on such process variables whose mean values are different for the normal status and the abnormal status, and it does not matter whether the variance of the process variables have changed in these two states.
5. Comparative analysis of experiments

In this section, the effectiveness of the proposed alarm design method will be verified through a numerical experiment (Experiment 1) and a real industrial case (Experiment 2). Experiment 1 assumes that the probability distribution of the process variable $x(t)$ is partially unknown, two situations of dynamic alarm evidence updating are considered in this experiment: the relevant fusion parameters are given subjectively by expert knowledge, or the calculation of the reliability factor and optimization of the importance weight are adopted respectively when the fusion parameters are not properly selected. Besides, Experiment 2 is a real case of oil pipeline leakage, the process variable $x(t)$ is the difference between the outlet fluid quality and the inlet fluid quality of the pipeline. Due to the lack of understanding of the complex fluid motion principle and unknown interference in the observation environment, the probability distribution of $x(t)$ is almost unknown.

In addition, in numerical experiment and real industrial case, the proposed method in this paper is compared with the traditional alarm methods such as combined on and off-delay timer method under various delay steps (ADT) and moving average filter method in different filtering orders (MAF), and the initial conditional evidence updating method (CEU) in [16]. The experimental results show that the proposed method has a better effect than the traditional methods and the CEU method in reducing FAR and MAR. At the same time, it also shows that when the relevant fusion parameters given by expert knowledge are not suitable, the performance of the proposed alarm design method can be further improved by adjusting these fusion parameters.

5.1. Experiment 1

In order to construct a sample with a partially unknown probability distribution, this experiment assumes that the process variable $x(t)$ follows a piecewise white Gaussian stochastic process: $x(t) \sim N(\mu_1, \sigma_1^2), \mu_1 \in [0.2, 0.3], \sigma_1 \in [1.5, 1.6]$ in the normal state and $x(t) \sim N(\mu_2, \sigma_2^2), \mu_2 \in [1.2, 1.5], \sigma_2 \in [1.5, 1.6]$ in the abnormal state. The data characteristics of normal data and abnormal data of the sampling sequence of the process variable $x(t)$ is shown in Fig. 2, $x(t)$ changed state at $t_0 = 1000h$. It can be seen from Fig. 2 that the partial unknown of the probability distribution of process variable $x(t)$ is specifically manifested in that the sampled data at each moment has a large variation range, and at the same time, the sampled data at the preceding and following moments has a high degree of coincidence. There will be great uncertainty in the classical alarm design method by judging whether the process variable $x(t)$ exceeds the alarm threshold $x_{tp}$.

The information of process variable is converted into alarm evidence by the method in Section 4.1. That is, the 2000 sets of data generated by piecewise white Gaussian stochastic process are used as training samples, the reference values set of input $x(t)$ is set to $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$, and the reference values set of output $y(t)$ is set to $\{0, 1\}$ according to expert knowledge. Based on these training samples, the comprehensive similarity distribution is used to statistically construct a sample reference value cast point table as shown in Table 4, and then a referential evidential matrix table is constructed, as shown in Table 5.

Continue to use these 2000 sets of data, according to the contents in Section 4.1, the sample reference value cast point...
of sample data. Here, Samplereferencevaluecastpointtable.

Table 4
Sample reference value cast point table.

<table>
<thead>
<tr>
<th>y(t)</th>
<th>x(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>−5</td>
<td>2.53</td>
</tr>
<tr>
<td>Y₁</td>
<td>Y₂</td>
</tr>
<tr>
<td>Total</td>
<td>2.53</td>
</tr>
</tbody>
</table>

The overlap between Figs. 3 and 4, the overlap between current alarm evidence and historical global alarm evidence is shown in Fig. 4, and the FAR and the MAR are 12.7% and 13.6% respectively under the alarm criterion. At the same time, it can be clearly seen from the comparison of Figs. 3 and 4, the overlap between m₁₁(NA) and m₁₁(A) in Fig. 3 is less than the overlap in Fig. 4. Obviously, through the dynamic adjustment of parameters (r, w), the fusion result m₁ is more strongly supports the true state of the process variable X(t). Therefore, the FAR and MAR of the proposed method under dynamic changes of the fusion parameters must be lower than the fusion parameters are fixed, and the final experimental results also prove this point. So when the relevant fusion parameters given by expert knowledge are not suitable, the performance of the proposed method can be further improved by adjusting these fusion parameters.

To fully demonstrate the effectiveness of the proposed method, using the piecewise white Gaussian stochastic process to generate 200 sets of test samples, performing 200 test experiments on X(t), and calculating the mean values of FAR, MAR and AAD of the proposed UER method, the alarm delay timer method under various delay steps (ADT), moving average filter method in different filtering orders (MAF), and the initial conditional evidence updating method (CEU) in [16] under the optimal alarm

2000 sets of data as test samples through piecewise white Gaussian stochastic process, formulas (21) and (22) are used to obtain the current alarm evidence mₙ based on the referential evidential matrix table; Then combining with optimized importance weight w₁ = 0.4412 and w₂ = 0.1471, formulas (27) to (32) are used to calculate the relevant reliability factors r₁ and r₁₋₁. On this basis, the global alarm evidence m₁₁ at each moment is obtained according to formula (23), Fig. 3 shows m₁₁(NA) and m₁₁(A) of the global alarm evidence m₁₁. Finally, the alarm decision is made according to the alarm criterion, and the FAR and the MAR are 2.7% and 1.6% respectively. As a contrast, another experiment is carried out on the proposed method without dynamic change of the fusion parameters (r, w). Repeating the above experimental steps, when using formula (23) to obtain the global alarm evidence m₁₁, the importance weight and the reliability factor of relevant alarm evidence are set to w₁ = w₂ = 1 and r₁₋₁ = r₂ = 0.9 respectively according to expert knowledge, as shown in Fig. 4, and the FAR and the MAR are 12.7% and 13.6% respectively under the alarm criterion.
threshold \( x_{opt} \) respectively, Tables 7 and 8 show the statistical results of the \( m_l(FAR) \), \( m_l(MAR) \) and AAD. Fig. 5 visually draws the line graphs about the synthesis \( g = (m_l(FAR)^2 + m_l(MAR)^2)^{0.5} \) about the \( m_l(FAR) \) and \( m_l(MAR) \) and AAD under those different methods. In addition, since the UER and CEU methods do not need to consider the delay steps or filtering orders, those two methods are represented by a smooth straight line in Fig. 5 respectively.

It can be seen from Fig. 5, Tables 7 and 8 that the FAR and MAR of the proposed UER method are the lowest when the probability distribution of process variables is partially unknown. On the one hand, there is no information loss in the process of...
Table 7
Comparison of performance evaluation indicators of UER/CEU/ADT.

<table>
<thead>
<tr>
<th>UER</th>
<th>CEU</th>
<th>ADT</th>
</tr>
</thead>
<tbody>
<tr>
<td>order = 3</td>
<td>order = 4</td>
<td>order = 5</td>
</tr>
<tr>
<td>(x_{op})</td>
<td>1.0</td>
<td>0.79</td>
</tr>
<tr>
<td>(m(FAR))</td>
<td>1.23%</td>
<td>5.63%</td>
</tr>
<tr>
<td>(m(MAR))</td>
<td>2.65%</td>
<td>4.26%</td>
</tr>
<tr>
<td>AAD</td>
<td>19.03</td>
<td>13.7</td>
</tr>
</tbody>
</table>

Table 8
Comparison of performance evaluation indicators of UER/CEU/MAF.

<table>
<thead>
<tr>
<th>UER</th>
<th>CEU</th>
<th>MAF</th>
</tr>
</thead>
<tbody>
<tr>
<td>step = 3</td>
<td>step = 4</td>
<td>step = 5</td>
</tr>
<tr>
<td>(x_{op})</td>
<td>1.0</td>
<td>0.79</td>
</tr>
<tr>
<td>(m(FAR))</td>
<td>1.23%</td>
<td>5.63%</td>
</tr>
<tr>
<td>(m(MAR))</td>
<td>2.65%</td>
<td>4.26%</td>
</tr>
<tr>
<td>AAD</td>
<td>19.03</td>
<td>13.7</td>
</tr>
</tbody>
</table>

Converting the process variable into alarm evidence through the referential evidential matrix table, which lays a good foundation for the subsequent fusion of corresponding alarm evidence. On the other hand, in the process of dynamic fusion of alarm evidence, the ER rule and the optimal design of the relevant fusion parameters are used to comprehensively consider the historical and current alarm evidence, so that the obtained current global alarm evidence is the most true and exhaustive response to process variable. Correspondingly, due to the limitation of the filtering orders and delay steps used in the data preprocessing stage of the classical alarm design methods such as ADT and MAF methods, the original information loss of the process variable is too large, eventually the FAR and MAR are too high; similarly, the information loss during the alarm evidence conversion process is large because the fuzzy membership function is a non-continuous function, which ultimately leads to the poor effect of the CEU method.

Note that as the filtering orders of the MAF method increases from step = 3 to step = 14, the AAD increases from 3.4 to 19.24. When step = 14, the AAD = 19.24 is equivalent to the UER
method, but $m$(FAR) = 10.28% and $m$(MAR) = 8.46% of the former are much higher than the $m$(FAR) = 1.23% and $m$(MAR) = 2.65% of the latter. Similarly, the $m$(FAR) = 1.28% of the ADT method when order = 8 is the closest to the proposed method in this paper, but AAD = 98.73, which is almost impossible to receive in actual industrial scenarios. It can be seen that the proposed UER method greatly reduces the FAR and MAR of the designed alarm without making the AAD higher, and finally achieves a comprehensive tradeoff between the sensitivity and accuracy.

5.2. Experiment 2

In this actual case, the length of the liquefied petroleum gas (LPG) pipeline detected reached 100 kilometers [25]. Mass flowmeters were set at the inlet and outlet of the pipeline respectively to collect flow data, and the sampling period were $h$ = 10 s. Opened the valve for a typical leak test lasting 23.62 h, the outlet flow and inlet flow during the whole test period were expressed as $f_1(t)$ and $f_0(t)$ (unit: kg) respectively. With a total of 8505 sampling points, the leakage time period was $t$ = 122h to $t$ = 2209h, which lasted 5.8 h in total as shown in Fig. 6. Theoretically, $f_1(t)$-$f_0(t)$ = 0 indicates the normal state, and $f_1(t)$-$f_0(t)$<0 indicates the abnormal state (that is, the pipeline leaks), here $x(t) = f_1(t)$-$f_0(t)$ is used as the process variable in the designed alarm.

However, there is usually an inevitable deviation between theoretical calculation and engineering practice as shown in Fig. 7, when the pipeline does not leak, $x(t)$ may be less than 0, so it is unreasonable to set the alarm threshold to $x_{tp} = 0$. The reasons for this deviation are difficult to quantify, such as fluid fluctuations, temperature and pressure changes, and turbulence, which all may cause this phenomenon randomly and accidentally. In addition, the sampled data is also contaminated by various unknown noise and interference. In short, due to these various possible reasons, the mean values and variance of $x(t)$ under normal and abnormal status have changed and it is impossible to model statistical characteristics only through sampled data. Therefore, the probability distribution of $x(t)$ here is almost completely unknown.

Using the historical data in Fig. 6 as a training sample, repeating the steps of Experiment 1 in Section 5.1 to obtain the optimal alarm threshold $x_{otp}$ of the ADT, MAF and the CEU methods, the importance weights $w_1 = 0.6837$ and $w_2 = 0.1560$ of the proposed UER methods. In addition, because the cost of conducting an oil pipeline leak test is too high, when verifying the effectiveness of the proposed method, the inlet flow $f_0(t)$ and the outlet flow $f_1(t)$ are added with 3% random disturbances as the test samples, namely $f_0'(t) = f_0(t)(1 + \vartheta)$ and $f_1'(t) = f_1(t)(1 + \vartheta)$, the random variables $\vartheta$ satisfies the uniform distribution $U(-0.03, 0.03)$. Based on this, 200 test sequences are generated, denoted as $x(t)_{8505}$. Then calculating the mean values of FAR, MAR and AAD of the proposed UER method, the ADT, MAF and CEU methods under the optimal alarm threshold $x_{otp}$ respectively, the comparison results are shown in Tables 9 and 10.

It can be seen from Tables 9 and 10, due to the low coincidence of the sampled data in the normal state and the abnormal state,
Fig. 7. The training data $x(t) = f_1(t) - f_0(t)$ calculated by Fig. 6.

Table 9 Comparison of performance evaluation indicators of UER/CEU/ADT.

<table>
<thead>
<tr>
<th></th>
<th>UER</th>
<th>CEU</th>
<th>ADT</th>
</tr>
</thead>
<tbody>
<tr>
<td>order = 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{otp}$</td>
<td>-0.72</td>
<td>-0.70</td>
<td>-0.70</td>
</tr>
<tr>
<td>$m(FAR)$</td>
<td>1.83%</td>
<td>3.63%</td>
<td>7.39%</td>
</tr>
<tr>
<td>$m(MAR)$</td>
<td>1.20%</td>
<td>1.26%</td>
<td>4.11%</td>
</tr>
<tr>
<td>AAD</td>
<td>0.16</td>
<td>1.17</td>
<td>3.38</td>
</tr>
</tbody>
</table>

Table 10 Comparison of performance evaluation indicators of UER/CEU/MAF.

<table>
<thead>
<tr>
<th></th>
<th>UER</th>
<th>CEU</th>
<th>MAF</th>
</tr>
</thead>
<tbody>
<tr>
<td>step = 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{otp}$</td>
<td>-0.72</td>
<td>-0.70</td>
<td>-0.70</td>
</tr>
<tr>
<td>$m(FAR)$</td>
<td>1.83%</td>
<td>3.63%</td>
<td>9.77%</td>
</tr>
<tr>
<td>$m(MAR)$</td>
<td>1.20%</td>
<td>1.26%</td>
<td>7.07%</td>
</tr>
<tr>
<td>AAD</td>
<td>0.16</td>
<td>1.17</td>
<td>0.98</td>
</tr>
</tbody>
</table>

the FAR, MAR and AAD of these four alarm design methods under this actual case are relatively low through compared with the numerical experiment in Section 5.1. In the horizontal comparison, compared with traditional ADT/MAF methods and the CEU method, the proposed UER method still have the lowest FAR, MAR and AAD. The advantages of ER rule in dealing with process variables whose probability distribution is partially unknown or completely unknown have been discussed in detail in the preface and will not be repeated here.

6. Conclusion

In order to deal with the generalized uncertainty of the process variable, this paper presents a purely data-driven approach without any probabilistic assumption based on the dynamic form of the evidence reasoning (ER) rule under the framework of Dempster–Shafer evidence theory (DST). Its main contributions are as follows: (1) constructs the referential evidential matrix to transform the raw information of process variable to the corresponding piece of alarm evidence and proposes the dynamic form of ER rule to combine the current and historical alarm evidence to generate the global alarm evidence for alarm decision; (2) introduces the forgetting strategy to realize the calculation of the reliability factors and uses genetic algorithm to realize the optimization of the importance weights respectively, so as to further improve the performance of the designed alarm system. Finally, numerical and real industrial case are given to show that the proposed method in this paper has a better performance than the classical optimization design methods and the initial conditional evidence updating method.

Assuredly, there are still some issues worth exploring in depth. For instance, this paper only considers the accuracy of the alarm by the FAR and the MAR to evaluate the alarm performance, and the sensitivity of the alarm is rarely considered. At the same time, the AAD also needs to be used as an evaluation indicator, because in actual engineering applications, AAD represents the degree of reaction of the designed alarm to changes in equipment status. Therefore, the FAR/MAR/AAD need to be comprehensively considered in order to achieve the best balance between the accuracy and sensitivity of the alarm in the future. Furthermore, this paper proposes the dynamic form of ER rule to recursively combine the current and historical alarm evidence. It is possible to consider using ER rule to fuse the alarm decision results obtained by multiple univariate alarms in some proper way, that is, to extend the design idea of univariate alarms to multivariate alarms, so as to give more precise judgment to the operating state of the equipment.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Acknowledgments

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jprocont.2021.07.006.

References